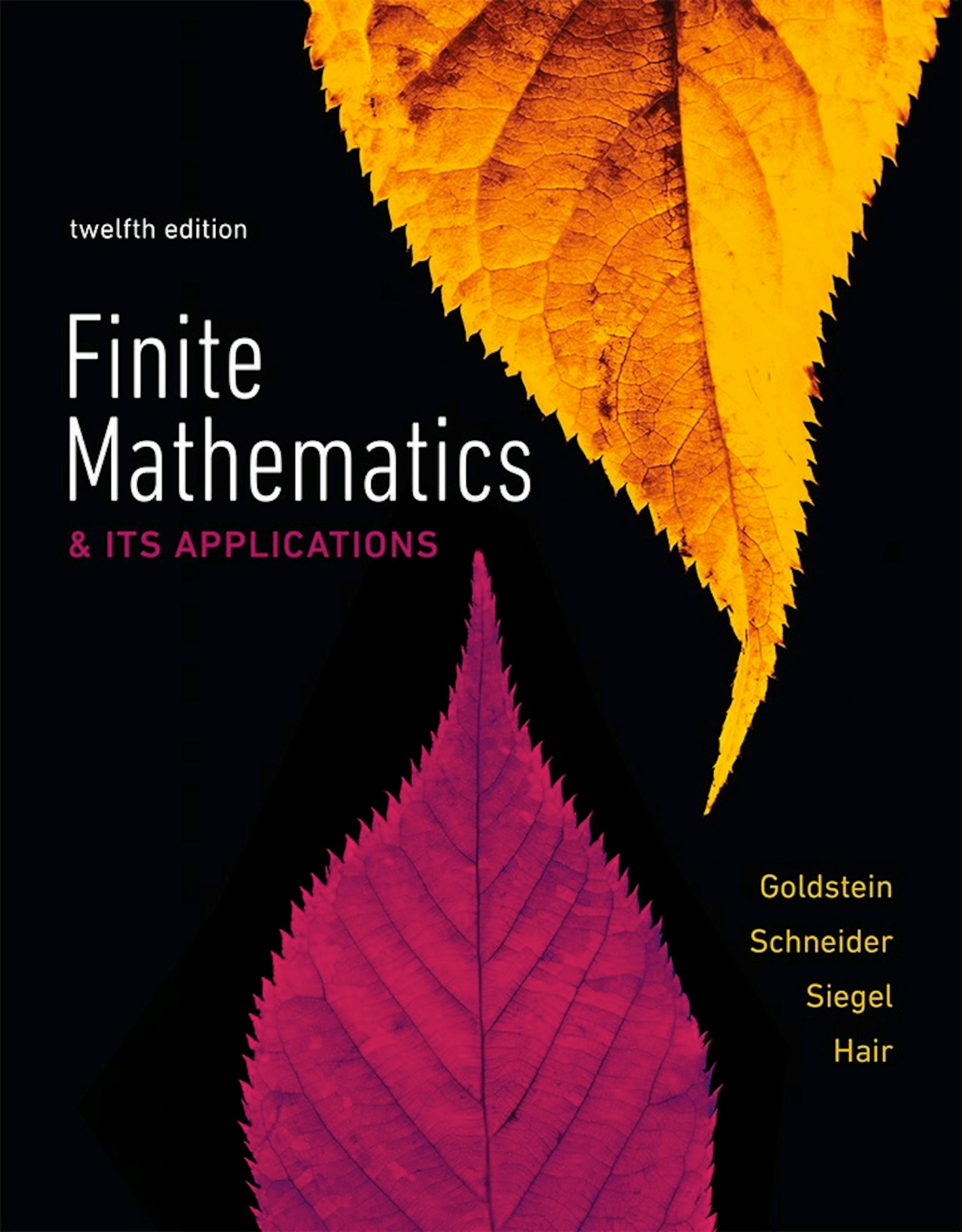
twelfth edition
Finite Mathematics
& ITS APPLICATIONS
Goldstein
Schneider
Siegel
Hair

MyMathLab®: Support You Need, When You Need It

MyMathLab is the world's leading online program in mathematics, integrating homework with support tools and tutorials in an easy-to-use format. MyMathLab helps you get up to speed on course material, visualize the content, and understand how math will play a role in your future career.

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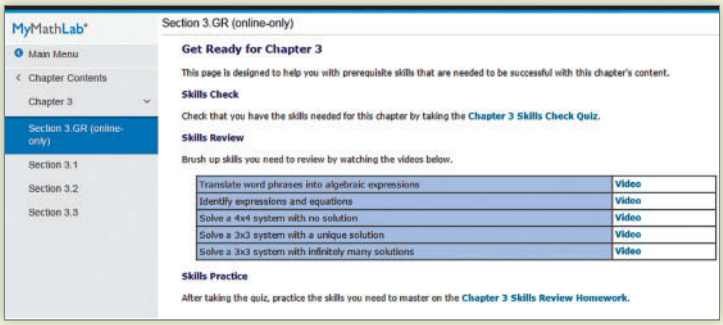
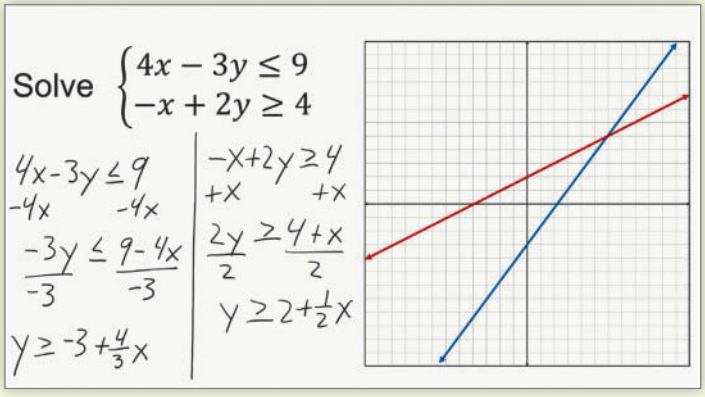


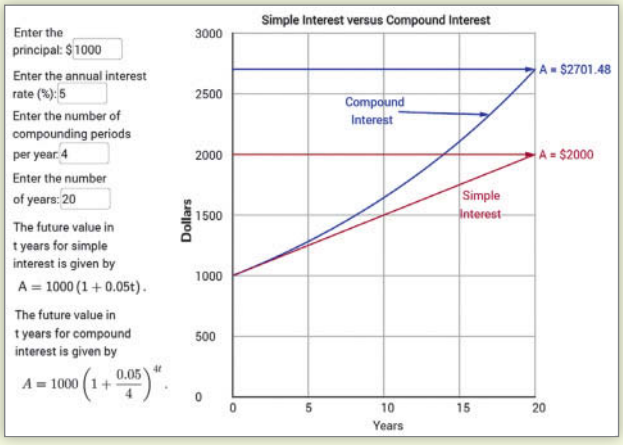


Tutorial Videos

Tutorial videos are available for every section of the textbook and cover key examples from the text. These videos are especially handy if you miss a lecture or just need another explanation.

Interactive Figures

Interactive Figures illustrate key concepts and help you visualize the math. MyMathLab includes assignable exercises that require use of Interactive Figures and instructional videos that explain the concept behind each figure.



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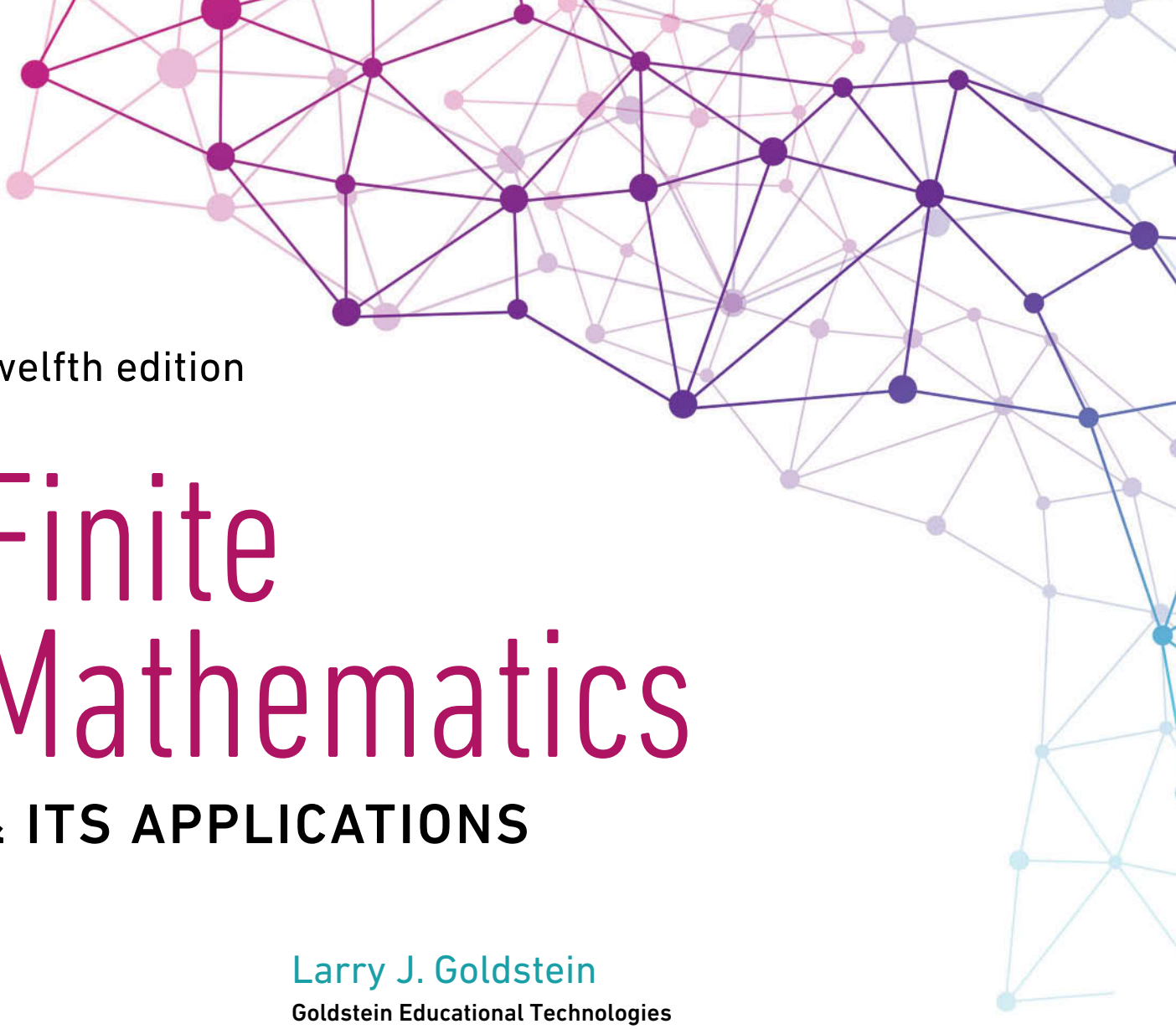

twelfth edition

Finite Mathematics

& ITS APPLICATIONS

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Pearson

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Contents

The book divides naturally into four parts. The first part consists of linear mathematics: linear equations, matrices, and linear programming (Chapters 1–4); the second part is devoted to probability and statistics (Chapters 5–7); the third part covers topics utilizing the ideas of the other parts (Chapters 8 and 9); and the fourth part explores key topics from discrete mathematics that are sometimes covered in the modern finite mathematics curriculum (Chapters 10–12).

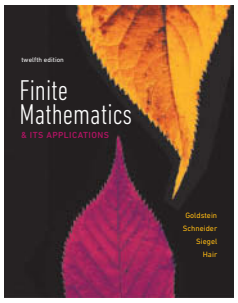
*www.pearsonhighered.com/mathstatsresources

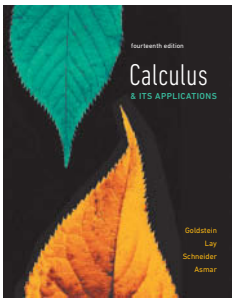
Preface

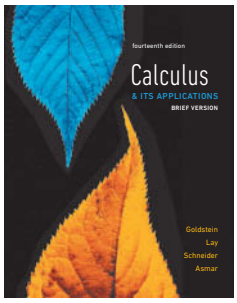
This work is the twelfth edition of our text for the finite mathematics course taught to first- and second-year college students, especially those majoring in business and the social and biological sciences. Finite mathematics courses exhibit tremendous diversity with respect to both content and approach. Therefore, in developing this book, we incorporated a wide range of topics from which an instructor may design a curriculum, as well as a high degree of flexibility in the order in which the topics may be presented. For the mathematics of finance, we even allow for flexibility in the approach of the presentation.

The Series

This text is part of a highly successful series consisting of three texts: *Finite Mathematics & Its Applications*, *Calculus & Its Applications*, and *Calculus & Its Applications, Brief Version*. All three titles are available for purchase in a variety of formats, including as an eBook within the MyMathLab online course.







Topics Included

This edition has more material than can be covered in most one-semester courses. Therefore, the instructor can structure the course to the students' needs and interests. The book divides naturally into four parts:

- Part One (Chapters 1–4) consists of linear mathematics: linear equations, matrices, and linear programming.
- Part Two (Chapters 5–7) is devoted to counting, probability, and statistics.
- Part Three (Chapters 8 and 9) covers topics utilizing the ideas of the other parts.
- Part Four (Chapters 10–12) explores key topics from discrete mathematics that are sometimes included in the modern finite mathematics curriculum.

Minimal Prerequisites

Because of great variation in student preparation, we keep formal prerequisites to a minimum. We assume only a first year of high school algebra, and we review, as needed, those topics that are typically weak spots for students.

New to This Edition

We welcome to this edition a new co-author, Steven Hair from Penn State University. Steve has brought a fresh eye to the content and to the MyMathLab course that accompanies the text.

We are grateful for the many helpful suggestions made by reviewers and users of the text. We incorporated many of these into this new edition. We also analyzed aggregated student usage and performance data from MyMathLab for the previous edition of this text. The results of this analysis helped improve the quality and quantity of exercises that matter the most to instructors and students. Additionally, we made the following improvements in this edition:

- **Help-Text Added.** We added blue "help text" next to steps within worked-out examples to point out key algebraic and numerical transitions.
- **Updated Technology.** We changed the graphing calculator screen captures to the more current TI-84 Plus CE format.The discussions of Excel now refer to Excel 2013 and Excel 2016.
- **Additional Exercises and Updated Data.** We have added or updated 440 exercises and have updated the real-world data appearing in the examples and exercises. The book now contains 3580 exercises and 370 worked-out examples.
- **Technology Solutions.** We added technology-based solutions to more examples to provide flexibility for instructors who incorporate technology. For instance, the section on the method of least-squares (1.4) now relies more on technology and less on complicated calculations. In Section 7.6, several examples now demonstrate how to compute the area under a normal curve using a graphing calculator, in addition to the table-based method. In the finance chapter, many TI-84 Plus TVM Solver screen captures accompany examples to confirm answers. Instructors have the option of using TVM Solver for financial calculations instead of complicated formulas.
- **Linear Inequalities Section Relocated.** We moved this section from 1.2 (in the 11e) to the beginning of the linear programming chapter (Ch. 3) in this edition. The move places the topic in the chapter where it is used. Also, the move allows us to use conventional names (such as *slope-intercept form*) in the section.
- **Improved Coverage of Counting Material.** In Chapter 5, we added several definitions and discussions to aid student comprehension of counting problems. We moved the definition of *factorials* to 5.4 and rewrote the permutation and combination formulas in 5.5 in terms of factorials. In 5.6, the *complement rule* for counting is now formally defined, and we have added a discussion of when addition, subtraction, and multiplication is appropriate for solving counting problems.
- **Section Added to the End of the Finance Chapter.** Titled "A Unifying Equation," this new section shows that the basic financial concepts can be described by a difference equation of the form $y_n = a \cdot y_{n-1} + b$, y_0 given, and that many of the calculations from the chapter can be obtained by solving this difference equation. Examples and exercises show that this difference equation also can be used to solve problems in the physical, biological, and social sciences. This section can be taught as a standalone section without covering the preceding sections of the finance chapter.
- **Revision of Logic Material.** We substantially revised Chapter 11 on logic to better meet student needs.We moved the definition of *logical equivalence* and De Morgan's laws from 11.4 to 11.2. By stating key ideas related to truth tables and implications in terms of logical equivalence, students will be better equipped to understand these concepts. To remove confusion between the inclusive and exclusive "or" statements, we removed the word "either" from inclusive "or" statements in English. In 11.4, we added the definition of the *inverse* of an implication. This is a key concept in the topic of implications and logical arguments. To help students understand when a logical argument is invalid, we expanded 11.5 to include more discussion of invalid arguments. Additionally, we added the fallacies of the inverse and converse, and two new examples where arguments are proven to be invalid.

- **Difference Equation Chapter Moved Online.** We moved former Chapter 11 online (relabeling it Chapter 12 in the process). The chapter is available directly to students at www.pearsonhighered.com/mathstatsresources and within MyMathLab. All support materials for the chapter appear online within MyMathLab. *Note:* The new section at the end of the finance chapter contains the fundamental concepts from the difference equation chapter.

New to MyMathLab

Many improvements have been made to the overall functionality of MyMathLab (MML) since the previous edition. However, beyond that, we have also invested in increasing and improving the content specific to this text.

- Instructors now have more exercises than ever to choose from in assigning homework. There are approximately 2540 assignable exercises in MML.
- We heard from users that the Annotated Instructor Edition for the previous edition required too much flipping of pages to find answers, so MML now contains a downloadable Instructor Answers document—*with all answers in one place*. (This augments the downloadable Instructor Solutions Manual, which contains all *solutions*.)
- Interactive Figures are now in HTML format (no plug-in required) and are supported by assignable exercises and tutorial videos.
- An Integrated Review version of the MML course contains pre-made quizzes to assess the prerequisite skills needed for each chapter, plus personalized remediation for any gaps in skills that are identified.
- New Setup & Solve exercises require students to show how they set up a problem as well as the solution, better mirroring what is required of students on tests.
- StatCrunch, a fully functional statistics package, is provided to support the statistics content in the course.
- MathTalk and StatTalk videos highlight applications of the content of the course to business. The videos are supported by assignable exercises.
- Study skills modules help students with the life skills that can make the difference between passing and failing.
- 110 new tutorial videos by Brian Rickard (University of Arkansas) were added to support student learning.
- Tutorial videos involving graphing calculators are now included within MML exercises to augment videos showing "by hand" methods. If you require graphing calculator usage for the course, your students will find these videos very helpful. (If you do not use calculators, you can hide these videos from students.)
- Graphing Calculator and Excel Spreadsheet Manuals, specific to this course, are now downloadable from MML.

Trusted Features

Though this edition has been improved in a variety of ways to reflect changing student needs, we have maintained the popular overall approach that has helped students be successful over the years.

Relevant and Varied Applications

We provide realistic applications that illustrate the uses of finite mathematics in other disciplines and everyday life. The variety of applications is evident in the Index of Applications at the end of the text. Wherever possible, we attempt to use applications to motivate the mathematics. For example, the concept of linear programming is introduced in Chapter 3 via a discussion of production options for a factory with labor limitations.

Plentiful Examples

The twelfth edition includes 370 worked examples. Furthermore, we include computational details to enhance comprehension by students whose basic skills are weak.

Knowing that students often refer back to examples for help, we built in fidelity between exercises and examples. In addition, students are given Now Try exercise references immediately following most examples to encourage them to check their understanding of the given example.

Exercises to Meet All Student Needs

The 3580 exercises comprise about one-quarter of the book—the most important part of the text, in our opinion. The exercises at the ends of the sections are typically arranged in the order in which the text proceeds, so that homework assignments may be made easily after only part of a section is discussed. Interesting applications and more challenging problems tend to be located near the ends of the exercise sets. Exercises have odd-even pairing, when appropriate. Chapter Review Exercises are designed to prepare students for end-of-chapter tests. Answers to the odd-numbered exercises, and all Chapter Review Exercises, are included at the back of the book.

Check Your Understanding Problems

The Check Your Understanding problems are a popular and useful feature of the book. They are carefully selected exercises located at the end of each section, just before the exercise set. Complete solutions follow the exercise set. These problems prepare students for the exercise sets beyond just covering simple examples. They give students a chance to think about the skills they are about to apply and reflect on what they've learned.

Use of Technology

We incorporated technology usage into the text in ways that provide you with flexibility, knowing that the course can vary quite a bit based on how technology is incorporated. Our basic approach in the text is to assume minimal use of technology and clearly label the opportunities to make it a greater part of the course. Many of the sections contain Incorporating Technology features that show how to use Texas Instruments graphing calculators, Excel spreadsheets, and Wolfram|Alpha. In addition, the text contains appendixes on the use of these technologies. Each type of technology is clearly labeled with an icon:

 (Graphing Calculator),

 (Spreadsheet),

WolframAlpha (Wolfram|Alpha)

In our discussions of graphing calculators, we specifically refer to the TI-84 Plus models, since these are the most popular graphing calculators. New to this edition, screen shots display the new color versions of the TI-84. Spreadsheets refer to Microsoft Excel 2016. The web application discussed is Wolfram|Alpha, which is an exceptionally fine and versatile product that is available online or on mobile devices for free or at low cost. We feel that Wolfram|Alpha is a powerful tool for learning and exploring mathematics, which is why we chose to include activities that use it. We hope that by modeling appropriate use of this technology, students will come to appreciate the application for its true worth.

End-of-Chapter Study Aids

Near the end of each chapter is a set of problems entitled Fundamental Concept Check Exercises that help students recall key ideas of the chapter and focus on the relevance of these concepts as well as prepare for exams. Each chapter also contains a two-column grid giving a section-by-section summary of key terms and concepts with examples. Finally, each chapter has Chapter Review Exercises that provide more practice and preparation for chapter-level exams.

Chapter Projects

Each chapter ends with an extended project that can be used as an in-class or out-of-class group project or special assignment. These projects develop interesting applications or enhance key concepts of the chapters.

Technology and Supplements

MyMathLab® Online Course (access code required)

Built around Pearson's best-selling content, MyMathLab is an online homework, tutorial, and assessment program designed to work with this text to engage students and improve results. MyMathLab can be successfully implemented in any classroom environment—lab-based, hybrid, fully online, or traditional. **By addressing instructor and student needs, MyMathLab improves student learning.**

Used by more than 37 million students worldwide, MyMathLab delivers consistent, measurable gains in student learning outcomes, retention, and subsequent course success. Visit www.mymathlab.com/results to learn more.

Preparedness

One of the biggest challenges in Finite Mathematics courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. Pearson offers a variety of content and course options to support students with just-in-time remediation and key-concept review.

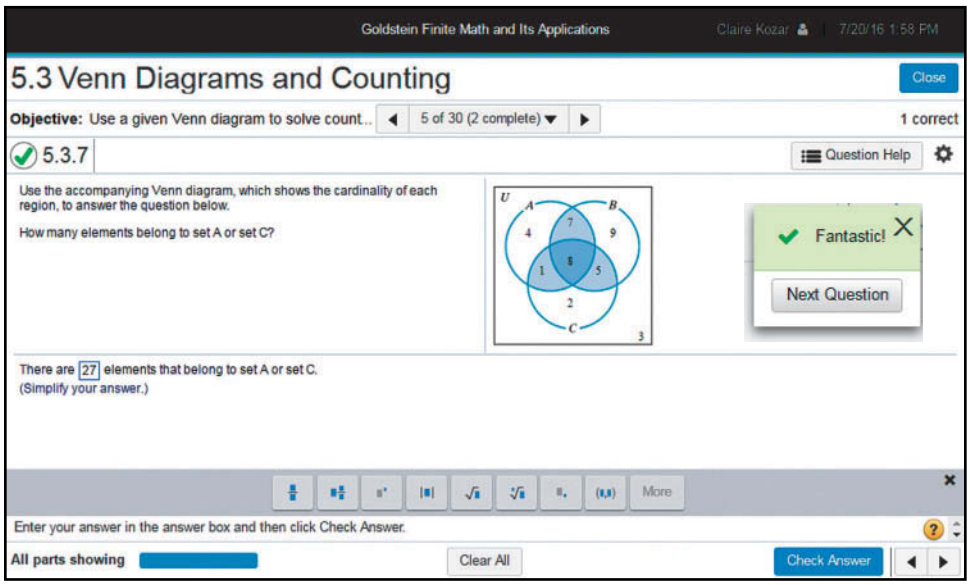
▲ Chapter 4 Skills Check Quiz
▲ Chapter 4 Skills Review Homework

- **Integrated Review Courses** can be used for just-in-time prerequisite review. These courses provide additional content on review topics, along with pre-made, assignable skill-check quizzes, personalized homework assignments, and videos integrated throughout the course.

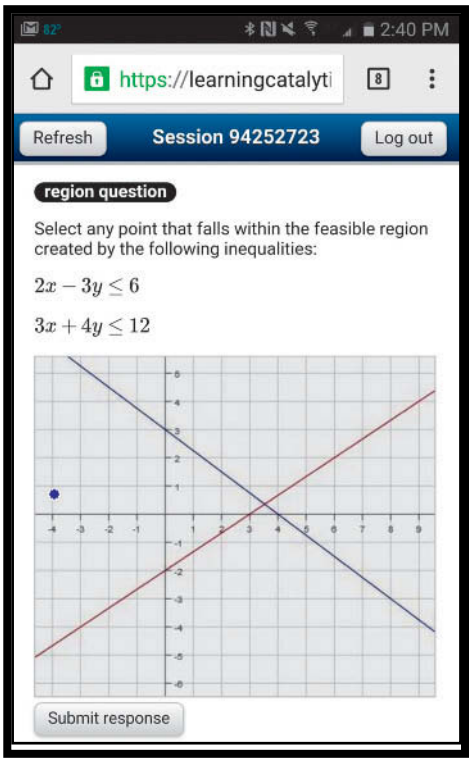
Motivation

Students are motivated to succeed when they're engaged in the learning experience and understand the relevance and power of mathematics. MyMathLab's online homework offers students immediate feedback and tutorial assistance that motivates them to do more, which means they retain more knowledge and improve their test scores.

- **Exercises with immediate feedback**—over 2540 assignable exercises—are based on the textbook exercises, and regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyMathLab provides helpful feedback when students enter incorrect answers and includes optional learning aids including Help Me Solve This, View an Example, videos, and an eText.

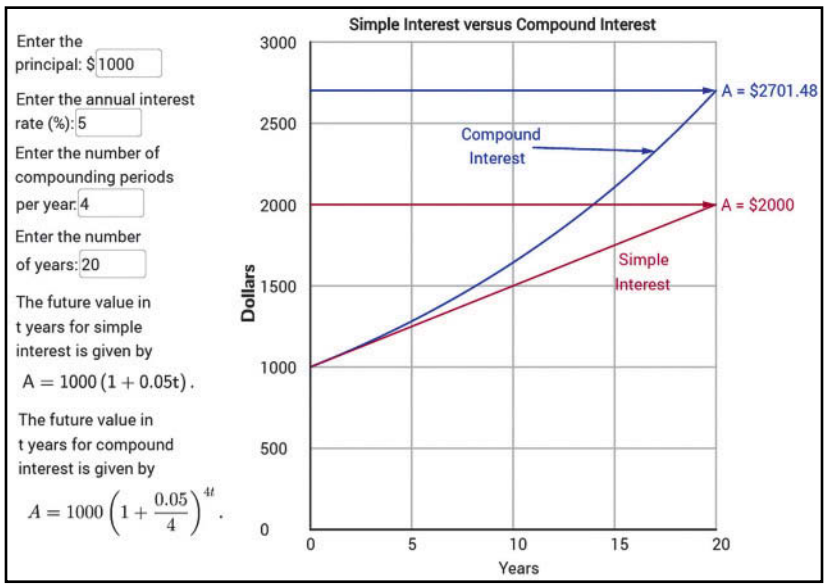


- **Setup and Solve Exercises** ask students to first describe how they will set up and approach the problem. This reinforces students' conceptual understanding of the process they are applying and promotes long-term retention of the skill.
- **MathTalk and StatTalk videos** connect the math to the real world (particularly business). The videos include assignable exercises to gauge students' understanding of video content.
- **Learning Catalytics™** is a student response tool that uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking. Learning Catalytics fosters student engagement and peer-to-peer learning with real-time analytics.



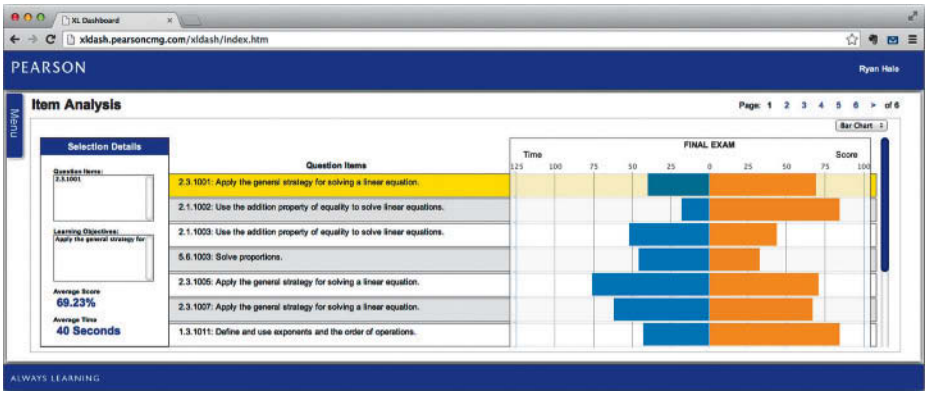
Learning and Teaching Tools

- **Interactive Figures** illustrate key concepts and allow manipulation for use as teaching and learning tools. MyMathLab includes assignable exercises that require use of figures and instructional videos that explain the concept behind each figure.



- **Instructional videos**—238 example-based videos—are available as learning aids within exercises and for self-study. The Guide to Video-Based Assignments makes it easy to assign videos for homework by showing which MyMathLab exercises correspond to each video.

- **Graphing Calculator videos** are available to augment "by hand" methods, allowing you to match the help that students receive to how graphing calculators are used in the course. Videos are available within select exercises and in the Multimedia Library.
- **Complete eText** is available to students through their MyMathLab courses for the lifetime of the edition, giving students unlimited access to the eText within any course using that edition of the textbook.
- **StatCrunch**, a fully functional statistics package, is provided to support the statistics content in the course.
- **Skills for Success Modules** help students with the life skills that can make the difference between passing and failing. Topics include "Time Management" and "Stress Management."
- **Excel Spreadsheet Manual,** specifically written for this course.
- **Graphing Calculator Manual,** specifically written for this course.
- **PowerPoint Presentations** are available for download for each section of the book.
- **Accessibility** and achievement go hand in hand. MyMathLab is compatible with the JAWS screen reader, and enables multiple-choice and free-response problem types to be read and interacted with via keyboard controls and math notation input. MyMathLab also works with screen enlargers, including ZoomText, MAGic, and SuperNova. And, all MyMathLab videos have closed-captioning. More information is available at http://mymathlab.com/accessibility.
- **A comprehensive gradebook** with enhanced reporting functionality allows you to efficiently manage your course.
  - **The Reporting Dashboard** provides insight to view, analyze, and report learning outcomes. Student performance data is presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.



  - **Item Analysis** tracks class-wide understanding of particular exercises so you can refine your class lectures or adjust the course/department syllabus. Just-in-time teaching has never been easier!

MyMathLab comes from an experienced partner with educational expertise and an eye on the future. Whether you are just getting started with MyMathLab, or have a question along the way, we're here to help you learn about our technologies and how to incorporate them into your course. To learn more about how MyMathLab helps students succeed, visit www.mymathlab.com or contact your Pearson rep.

MathXL® is the homework and assessment engine that runs MyMathLab. (MyMathLab is MathXL plus a learning management system.) MathXL access codes are also an option.

Student Solutions Manual

ISBN-10: 0-134-46344-7 | ISBN-13: 978-0-134-46344-5
Contains fully worked-out solutions to odd-numbered exercises. Available in print and downloadable from within MyMathLab.

Instructor Answers / Instructor Solutions Manual (downloadable)

ISBN-10: 0-134-46343-9 | ISBN-13: 978-0-134-46343-8
The Instructor Answers document contains a list of answers to all student edition exercises. The Instructor Solutions Manual contains solutions to all student edition exercises. Downloadable from the Pearson Instructor Resource Center www.pearsonhighered.com/irc, or from within MyMathLab.

TestGen (downloadable)

ISBN-10: 0-134-46346-3 | ISBN-13: 978-0-134-46346-9
TestGen enables instructors to build, edit, print, and administer tests using a bank of questions developed to cover all objectives in the text. TestGen is algorithmically based, allowing you to create multiple but equivalent versions of the same question or test. Instructors can also modify testbank questions or add new questions. The software and testbank are available to qualified instructors for download and installation from Pearson's online catalog www.pearsonhighered.com and from within MyMathLab.

PowerPoints

ISBN-10: 0-134-46407-9 | ISBN-13: 978-0-134-46407-7
Contains classroom presentation slides for this textbook featuring lecture content, worked-out examples, and key graphics from the text. Available to qualified instructors within MyMathLab or through the Pearson Instructor Resource Center www.pearsonhighered.com/irc.

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If you have any comments or suggestions, we would like to hear from you. We hope you enjoy using this book as much as we have enjoyed writing it.

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CHAPTER

1

Linear Equations and Straight Lines

Many applications considered later in this text involve linear equations and their geometric counterparts—straight lines. So let us begin by studying the basic facts about these two important notions.

1.1 Coordinate Systems and Graphs

Often, we can display numerical data by using a **Cartesian coordinate system** on either a line or a plane. We construct a Cartesian coordinate system on a line by choosing an arbitrary point O (the **origin**) on the line and a unit of distance along the line. We then assign to each point on the line a number that reflects its directed distance from the origin. Positive numbers refer to points on the right of the origin, negative numbers to points on the left. In Fig. 1, we have drawn a Cartesian coordinate system on the line and have labeled a number of points with their corresponding numbers. Each point on the line corresponds to a number (positive, negative, or zero).

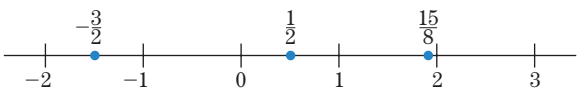


Figure 1

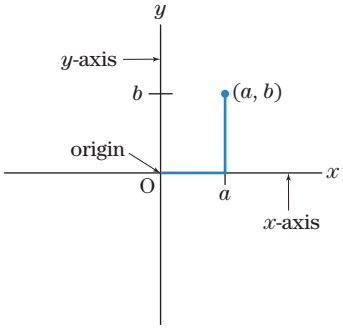


Figure 2

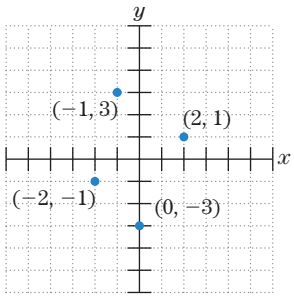
In a similar fashion, we can construct a Cartesian coordinate system to numerically locate points on a plane. Each point of the plane is identified by a pair of numbers (a, b). See Fig. 2. To reach the point (a, b), begin at the origin, move a units in the x direction (to the right if a is positive, to the left if a is negative), and then move b units in the y

direction (up if b is positive, down if b is negative). The numbers a and b are called, respectively, the ***x*-** and ***y*-coordinates** of the point.

EXAMPLE 1 **Plotting Points** Plot the following points:

(a) $(2, 1)$ **(b)** $(-1, 3)$ **(c)** $(-2, -1)$ **(d)** $(0, -3)$

SOLUTION



» Now Try Exercise 1

An equation in x and y is satisfied by the point (a, b) if the equation is true when x is replaced by a and y is replaced by b. This collection of points is usually a curve of some sort and is called the **graph of the equation**.

EXAMPLE 2 **Solution of an Equation** Are the following points on the graph of the equation $8x - 4y = 4$?

(a) $(3, 5)$ **(b)** $(5, 17)$

SOLUTION

(a)
$$\begin{aligned} 8x - 4y &= 4 && \text{Given equation} \\ 8 \cdot 3 - 4 \cdot 5 &\stackrel{?}{=} 4 && x = 3,\ y = 5 \\ 24 - 20 &\stackrel{?}{=} 4 && \text{Multiply.} \\ 4 &= 4 && \text{Subtract.} \end{aligned}$$

Since the equation is satisfied, the point $(3, 5)$ is on the graph of the equation.

(b)
$$\begin{aligned} 8x - 4y &= 4 && \text{Given equation} \\ 8 \cdot 5 - 4 \cdot 17 &\stackrel{?}{=} 4 && x = 5,\ y = 17 \\ 40 - 68 &\stackrel{?}{=} 4 && \text{Multiply.} \\ -28 &\stackrel{?}{=} 4 && \text{Subtract.} \end{aligned}$$

The equation is not satisfied, so the point $(5, 17)$ is *not* on the graph of the equation.

» Now Try Exercises 11 and 13

Linear Equations

A linear equation is an equation whose graph is a straight line. Figure 3 shows four examples of linear equations, along with their graphs and some points on their graphs.

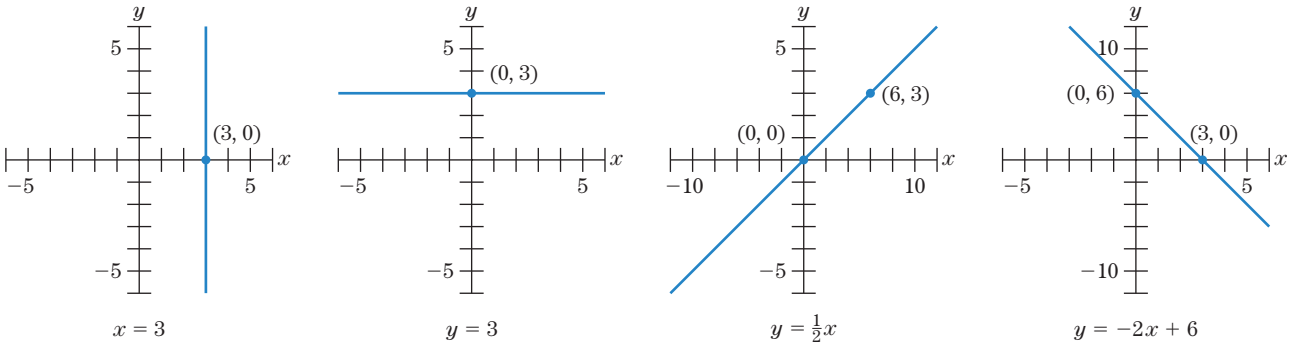


Figure 3 Four linear equations and their graphs

Intercepts

The intercepts of a line are the points where the line crosses the x- and y-axes. These points have 0 for at least one of their coordinates. For the graph of $y = -2x + 6$ in Fig. 3, the x-intercept is the point $(3, 0)$ and the y-intercept is the point $(0, 6)$.* The y-intercept of a line having an equation of the form $y = mx + b$ is the point $(0, b)$, since setting x equal to 0 gives y the value b. The x-intercept is the point having the solution of the equation $0 = mx + b$ as the first coordinate and 0 as the second coordinate.

Table 1 shows how to draw the graphs of the four types of linear equations shown in Fig. 3. The equations $y = b$ and $y = mx$ are actually special cases of $y = mx + b$.

Table 1 Graphs of Linear Equations

Equation	Description of Graph	How to Draw Graph
$x = a$	Vertical line through the point $(a, 0)$	Plot $(a, 0)$ and draw the vertical line through the point.
$y = b$	Horizontal line through the point $(0, b)$	Plot $(0, b)$ and draw the horizontal line through the point.
$y = mx$	Line through the origin	Draw the line through the origin and any other point on the graph.
$y = mx + b$; $m \neq 0, b \neq 0$	Line having two different intercepts	Draw the line through any two points (often the two intercepts) of the line.

General Form of a Linear Equation Any equation whose graph is a straight line can be written in the **general form**

$$cx + dy = e$$

where c, d, and e are constants and c and d are not both zero.

An equation in general form having $d \neq 0$ (that is, an equation in which y appears) can be solved for y. The resulting equation will have the form of one of the last three equations in Table 1. An equation in which y does not appear can be solved for x and the resulting equation will have the form of the first equation in Table 1.

EXAMPLE 3 **Graph of an Equation** Write the equation $x - 2y = 4$ in one of the forms shown in Table 1 and draw its graph.

SOLUTION Since y appears in the equation, solve for y.

$$\begin{aligned} x - 2y &= 4 && \text{Given equation} \\ -2y &= -x + 4 && \text{Subtract } x \text{ from both sides.} \\ y &= \tfrac{1}{2}x - 2 && \text{Divide both sides by } -2. \end{aligned}$$

Since the equation $y = \frac{1}{2}x - 2$ has the form of the last equation in Table 1, it can be graphed by finding its two intercepts and drawing the straight line through them.

*Intercepts are sometimes defined as numbers, such as x-intercept 3 and y-intercept 6. In this text, we define them as pairs of numbers, such as $(3, 0)$ and $(0, 6)$.

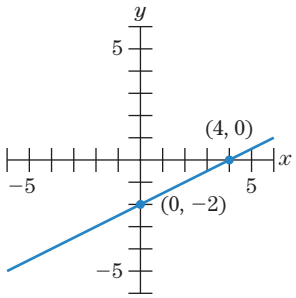


Figure 4 Graph of $x - 2y = 4$

The y-intercept is the point $(0, -2)$ since setting x equal to 0 gives y the value -2. The x-intercept is found by setting y equal to 0 and solving for x.

$$\begin{aligned} y &= \tfrac{1}{2}x - 2 && \text{Given equation} \\ 0 &= \tfrac{1}{2}x - 2 && \text{Set } y \text{ equal to 0.} \\ 2 &= \tfrac{1}{2}x && \text{Add 2 to both sides.} \\ x &= 4 && \text{Multiply both sides by 2. Rewrite.} \end{aligned}$$

Therefore, the x-intercept is the point $(4, 0)$.

The graph in Fig. 4 was obtained by plotting the intercepts $(4, 0)$ and $(0, -2)$ and drawing the straight line through them.

» Now Try Exercise 27

EXAMPLE 4 **Graph of an Equation** Write the equation $-2x + 3y = 0$ in one of the forms shown in Table 1 and draw its graph.

SOLUTION Since y appears in the equation, solve for y.

$$\begin{aligned} -2x + 3y &= 0 && \text{Given equation} \\ 3y &= 2x && \text{Add } 2x \text{ to both sides.} \\ y &= \tfrac{2}{3}x && \text{Divide both sides by 3.} \end{aligned}$$

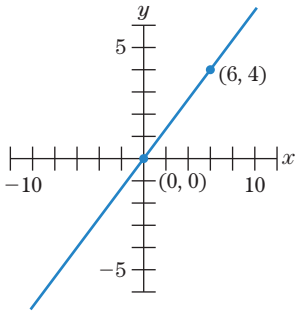


Figure 5 Graph of $-2x + 3y = 0$

Because the graph of the equation $y = \frac{2}{3}x$ passes through the origin, the point $(0, 0)$ is both the x-intercept and the y-intercept of the graph. In order to draw the graph, we must locate another point on the graph. Let's choose $x = 6$. Then $y = \frac{2}{3} \cdot 6 = 4$. Therefore, the point $(6, 4)$ is on the graph. The graph in Fig. 5 was obtained by plotting the points $(0, 0)$ and $(6, 4)$ and drawing the straight line through them.

» Now Try Exercise 19

The next example gives an application of linear equations.

EXAMPLE 5 **Linear Depreciation** For tax purposes, businesses must keep track of the current values of each of their assets. A common mathematical model is to assume that the current value y is related to the age x of the asset by a linear equation. A moving company buys a 40-foot van with a useful lifetime of 5 years. After x months of use, the value y, in dollars, of the van is estimated by the linear equation

$$y = 25{,}000 - 400x.$$

(a) Draw the graph of this linear equation.
(b) What is the value of the van after 5 years?
(c) When will the value of the van be \$15,000?
(d) What economic interpretation can be given to the y-intercept of the graph?

SOLUTION **(a)** The y-intercept is $(0, 25{,}000)$. To find the x-intercept, set $y = 0$ and solve for x.

$$\begin{aligned} 0 &= 25{,}000 - 400x && \text{Set } y = 0. \\ 400x &= 25{,}000 && \text{Add } 400x \text{ to both sides.} \\ x &= 62.5 && \text{Divide both sides by 400.} \end{aligned}$$

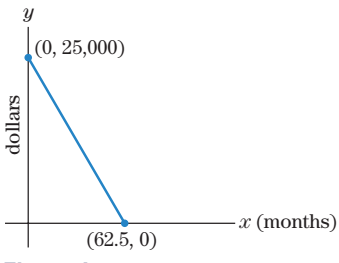


Figure 6

The x-intercept is $(62.5, 0)$. The graph of the linear equation is sketched in Fig. 6. Note how the value decreases as the age of the van increases. The value of the van reaches 0 after 62.5 months. Note also that we have sketched only the portion of the graph that has physical meaning—namely, the portion for x between 0 and 62.5.

(b) After 5 years (or 60 months), the value of the van is

$$y = 25{,}000 - 400(60) = 25{,}000 - 24{,}000 = 1000.$$

Since the useful life of the van is 5 years, this value represents the *salvage value* of the van.

(c) Set the value of y to 15,000, and solve for x.

$$\begin{aligned} 15{,}000 &= 25{,}000 - 400x && \text{Set } y = 15{,}000. \\ 400x + 15{,}000 &= 25{,}000 && \text{Add } 400x \text{ to both sides.} \\ 400x &= 10{,}000 && \text{Subtract } 15{,}000 \text{ from both sides.} \\ x &= 25 && \text{Divide both sides by } 400. \end{aligned}$$

The value of the van will be \$15,000 after 25 months.

(d) The y-intercept corresponds to the value of the van at $x = 0$ months—that is, the initial value of the van, \$25,000. **»» Now Try Exercise 41**

INCORPORATING TECHNOLOGY

Appendix B contains instructions for TI-84 Plus calculators. (For the specifics of other calculators, consult the guidebook for the calculator.) The appendix shows how to obtain the graph of a linear equation of the form $y = mx + b$, find coordinates of points on the line, and determine intercepts. Vertical lines can be drawn with the Vertical command from the DRAW menu. To draw the vertical line $x = k$, go to the home screen, press 2nd [DRAW] **4** to display the word Vertical, type in the value of k, and press ENTER.

WolframAlpha Appendix D contains an introduction to Wolfram|Alpha.

Straight lines can be drawn with instructions of the following forms:

plot ax + by = c; plot y = ax + b; plot x = a

If a phrase of the form **for x from x_1 to x_2** is appended to the instruction, only the portion of the line having x-values from x_1 to x_2 will be drawn.

An equation of the form $ax + by = c$, with $b \neq 0$, can be converted to the form $y = mx + b$ with the instruction **solve ax + by = c for y**.

The intercepts of an equation can be found with an instruction of the form **intercepts [equation]**. An expression in x can be evaluated at $x = a$ with an instruction of the form **evaluate [expression] at x = a**. For instance, the instruction

evaluate 2500 − 400x at x = 5

gives the result 500.

Check Your Understanding 1.1

Solutions can be found following the section exercises.

1. Plot the point (500, 200).

2. Is the point $(4, -7)$ on the graph of the linear equation $2x - 3y = 1$? Is the point $(5, 3)$?

EXERCISES 1.1

In Exercises 1–8, plot the given point.

1. $(2, 3)$

2. $(-1, 4)$

3. $(0, -2)$

4. $(2, 0)$

5. $(-2, 1)$

6. $\left(-1, -\frac{5}{2}\right)$

7. $(-20, 40)$

8. $(25, 30)$

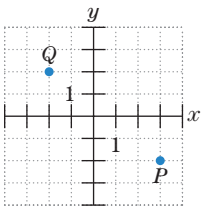


Figure 7

9. What are the coordinates of the point Q in Fig. 7?

10. What are the coordinates of the point P in Fig. 7?

In Exercises 11–14, determine whether the point is on the graph of the equation $-2x + \frac{1}{3}y = -1$.

11. $(1, 3)$ **12.** $(2, 6)$ **13.** $(\frac{1}{2}, 3)$ **14.** $(\frac{1}{3}, -1)$

In Exercises 15–18, each linear equation is in the form $y = mx + b$. Identify m and b.

15. $y = 5x + 8$ **16.** $y = -2x - 6$

17. $y = 3$ **18.** $y = \frac{2}{3}x$

In Exercises 19–22, write each linear equation in the form $y = mx + b$ or $x = a$.

19. $14x + 7y = 21$ **20.** $x - y = 3$

21. $3x = 5$ **22.** $-\frac{1}{2}x + \frac{2}{3}y = 10$

In Exercises 23–26, find the x-intercept and the y-intercept of each line.

23. $y = -4x + 8$ **24.** $y = 5$

25. $x = 7$ **26.** $y = -8x$

In Exercises 27–34, graph the given linear equation.

27. $y = \frac{1}{3}x - 1$ **28.** $y = 2x$ **29.** $y = \frac{5}{2}$

30. $x = 0$ **31.** $3x + 4y = 24$ **32.** $x + y = 3$

33. $x = -\frac{5}{2}$ **34.** $\frac{1}{2}x - \frac{1}{3}y = -1$

35. Which of the following equations describe the same line as the equation $2x + 3y = 6$?

(a) $4x + 6y = 12$ **(b)** $y = -\frac{2}{3}x + 2$ **(c)** $x = 3 - \frac{3}{2}y$

(d) $6 - 2x - y = 0$ **(e)** $y = 2 - \frac{2}{3}x$ **(f)** $x + y = 1$

36. Which of the following equations describe the same line as the equation $\frac{1}{2}x - 5y = 1$?

(a) $2x - \frac{1}{5}y = 1$ **(b)** $x = 5y + 2$

(c) $2 - 5x + 10y = 0$ **(d)** $y = .1(x - 2)$

(e) $10y - x = -2$ **(f)** $1 + .5x = 2 + 5y$

37. Each of the lines L_1, L_2, and L_3 in Fig. 8 is the graph of one of the equations (a), (b), and (c). Match each of the equations with its corresponding line.

(a) $x + y = 3$ **(b)** $2x - y = -2$ **(c)** $x = 3y + 3$

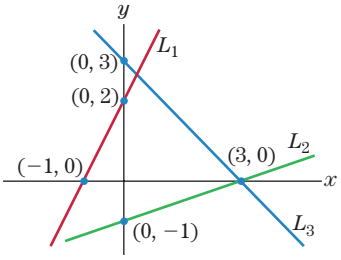


Figure 8

38. Which of the following equations is graphed in Fig. 9?

(a) $x + y = 3$ **(b)** $y = x - 1$ **(c)** $2y = x + 3$

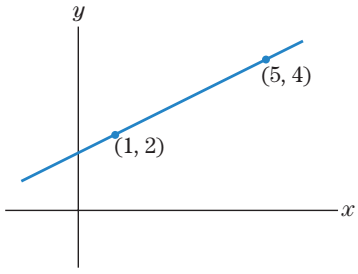


Figure 9

39. Heating Water The temperature of water in a heating tea kettle rises according to the equation $y = 30x + 72$, where y is the temperature (in degrees Fahrenheit) x minutes after the kettle was put on the burner.

(a) What physical interpretation can be given to the y-intercept of the graph?

(b) What will the temperature of the water be after 3 minutes?

(c) After how many minutes will the water be at its boiling point of 212°?

40. Life Expectancy The average life expectancy y of a person born x years after 1960 can be approximated by the linear equation $y = \frac{1}{6}x + 70$.

(a) What interpretation can be given to the y-intercept of the graph?

(b) In what year did people born that year have an average life expectancy of 75 years?

(c) What is the average life expectancy of people born in 1999?

41. Cigarette Consumption The worldwide consumption of cigarettes has been increasing steadily in recent years. The number of trillions of cigarettes, y, purchased x years after 1960, is estimated by the linear equation $y = .075x + 2.5$.

(a) Draw the graph of this linear equation.

(b) What interpretation can be given to the y-intercept of the graph?

(c) When were there 4 trillion cigarettes sold?

(d) If this trend continues, how many cigarettes will be sold in the year 2024?

42. Ecotourism Income In a certain developing country, ecotourism income has been increasing in recent years. The income y (in thousands of dollars) x years after 2000 can be modeled by $y = 1.15x + 14$.

(a) Draw the graph of this linear equation.

(b) What interpretation can be given to the y-intercept of this graph?

(c) When was there $20,000 in ecotourism income?

(d) If this trend continues, how much ecotourism income will there be in 2022?

43. Insurance Rates Yearly car insurance rates have been increasing steadily in the last few years. The rate y (in dollars) for a small car x years after 1999 can be modeled by $y = 23x + 756$.

(a) Draw the graph of this linear equation.

(b) What interpretation can be given to the y-intercept of this graph?

(c) What was the yearly rate in 2007?

(d) If this trend continues, when will the yearly rate be $1308?

44. Simple Interest If \$1000 is deposited at 3% simple interest, the balance y after x years will be given by the equation $y = 30x + 1000$.
(a) Draw the graph of this linear equation.
(b) Find the balance after two years.
(c) When will the balance reach \$1180?

45. College Freshmen The percentage, y, of college freshmen who entered college intending to major in general biology increased steadily from the year 2000 to the year 2014 and can be approximated by the linear equation $y = .2x + 4.1$ where x represents the number of years since 2000. Thus, $x = 0$ represents 2000, $x = 1$ represents 2001, and so on. (*Source: The American Freshman: National Norms.*)
(a) What interpretation can be given to the y-intercept of the graph of the equation?
(b) In 2014, approximately what percent of college freshmen intended to major in general biology?
(c) In what year did approximately 5.5% of college freshmen intend to major in general biology?

46. College Freshmen The percentage, y, of college freshmen who smoke cigarettes decreased steadily from the year 2004 to the year 2014 and can be approximated by the linear equation $y = -.46x + 6.32$ where x represents the number of years since 2004. Thus, $x = 0$ represents 2004, $x = 1$ represents 2005, and so on. (*Source: The American Freshman: National Norms.*)
(a) What interpretation can be given to the y-intercept of the graph of the equation?
(b) In 2014, approximately what percent of college freshmen smoked?
(c) In what year did approximately 2.6% of college freshmen smoke?

47. College Tuition Average tuition (including room and board) for all institutions of higher learning in year x can be approximated by $y = 461x + 16{,}800$ dollars, where $x = 0$ corresponds to 2004, $x = 1$ corresponds to 2005, and so on. (*Source:* U.S. National Center of Education Statistics.)
(a) Approximately what was the average tuition in 2011?
(b) Assuming that the formula continues to hold, when will the average tuition exceed \$25,000?

48. Bachelor's Degrees The number of bachelor's degrees conferred in mathematics and statistics in year x can be approximated by $y = 667x + 12{,}403$, where $x = 0$ corresponds to 2003, $x = 1$ corresponds to 2004, and so on. (*Source:* U.S. National Center of Education Statistics.)
(a) Approximately how many bachelor's degrees in mathematics and statistics were awarded in 2007?
(b) Assuming that the model continues to hold, approximately when will the number of bachelor's degrees in mathematics and statistics awarded exceed 25,000?

49. Find an equation of the line having x-intercept (16, 0) and y-intercept (0, 8).

50. Find an equation of the line having x-intercept (.6, 0) and y-intercept (0, .9).

51. Find an equation of the line having y-intercept (0, 5) and x-intercept (4, 0).

52. Find an equation of the line having x-intercept (5, 0) and parallel to the y-axis.

53. What is the equation of the x-axis?

54. Can a line other than the x-axis have more than one x-intercept?

55. What is the general form of the equation of a line that is parallel to the y-axis?

56. What is the general form of the equation of a line that is parallel to the x-axis?

In Exercises 57–60, find a general form of the given equation.

57. $y = 2x + 3$ **58.** $y = 3x - 4$

59. $y = -\frac{2}{3}x - 5$ **60.** $y = 4x - \frac{5}{6}$

61. Show that the straight line with x-intercept $(a, 0)$ and y-intercept $(0, b)$, where a and b are not zero, has $bx + ay = ab$ as a general form of its equation.

62. Use the result of Exercise 61 to find a general form of the equation of the line having x-intercept (5, 0) and y-intercept (0, 6).

In Exercises 63–70, give the equation of a line having the stated property. *Note:* There are many answers to each exercise.

63. x-intercept (9, 0)

64. y-intercept (0, 10)

65. passes through the point (−2, 5)

66. passes through the point (3, −3)

67. crosses the positive part of the y-axis

68. passes through the origin

69. crosses the negative part of the x-axis

70. crosses the positive part of the x-axis

71. The lines with equations $y = \frac{2}{3}x - 2$ and $y = -4x + c$ have the same x-intercept. What is the value of c?

72. The lines with equations $6x - 3y = 9$ and $y = 4x + b$ have the same y-intercept. What is the value of b?

TECHNOLOGY EXERCISES

In Exercises 73–76, (a) graph the line, (b) use the utility to determine the two intercepts, (c) use the utility to find the y-coordinate of the point on the line with x-coordinate 2.

73. $y = -3x + 6$ **74.** $y = .25x - 2$

75. $3y - 2x = 9$ **76.** $2y + 5x = 8$

In Exercises 77 and 78, determine an appropriate window, and graph the line.

77. $2y + x = 100$

78. $x - 3y = 60$

Solutions to Check Your Understanding 1.1

1. Because the numbers are large, make each hatchmark correspond to 100. Then the point (500, 200) is found by starting at the origin, moving 500 units to the right and 200 units up (Fig. 10 on the next page).

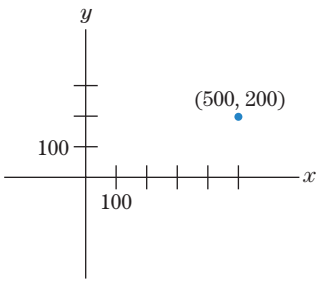


Figure 10

2.

$$\begin{aligned} 2x - 3y &= 1 && \text{Given equation} \\ 2(4) - 3(-7) &\stackrel{?}{=} 1 && x = 4,\ y = -7 \\ 29 &\stackrel{?}{=} 1 && \text{False} \end{aligned}$$

Since the equation is not satisfied, $(4, -7)$ is not on the graph.

$$\begin{aligned} 2x - 3y &= 1 && \text{Given equation} \\ 2(5) - 3(3) &\stackrel{?}{=} 1 && x = 5,\ y = 3 \\ 1 &= 1 && \text{True} \end{aligned}$$

Since the equation is satisfied, $(5, 3)$ is on the graph.

1.2 The Slope of a Straight Line

In this section, we consider only lines whose equations can be written in the form $y = mx + b$. Geometrically, this means that we will consider only nonvertical lines. Slope is not defined for vertical lines.

DEFINITION Given a nonvertical line L with equation $y = mx + b$, the number m is called the **slope** of L. That is, the slope is the coefficient of x in the equation of the line. The equation is called the **slope–intercept** form of the equation of the line.

EXAMPLE 1 **Finding the Slope of a Line from its Equation** Find the slopes of the lines having the following equations:

(a) $y = 2x + 1$ **(b)** $y = -\frac{3}{4}x + 2$ **(c)** $y = 3$ **(d)** $-8x + 2y = 4$

SOLUTION

(a) $m = 2$.

(b) $m = -\frac{3}{4}$.

(c) When we write the equation in the form $y = 0 \cdot x + 3$, we see that $m = 0$.

(d) First, write the equation in slope–intercept form.

$$\begin{aligned} -8x + 2y &= 4 && \text{Given equation} \\ 2y &= 8x + 4 && \text{Add } 8x \text{ to both sides.} \\ y &= 4x + 2 && \text{Divide both sides by 2.} \end{aligned}$$

Thus, $m = 4$. **»» Now Try Exercise 1**

The definition of the slope is given in terms of an equation of the line. There is an alternative equivalent definition of *slope*.

DEFINITION Alternative Definition of Slope Let L be a line passing through the points (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$. Then, the slope of L is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}. \tag{1}$$

That is, the slope is the difference in the y-coordinates divided by the difference in the x-coordinates, with both differences formed in the same order. *Note:* x_1 is pronounced "x sub 1."

Before proving this definition equivalent to the first one given, let us show how it can be used.

EXAMPLE 2 **Finding the Slope of a Line from Two Points** Find the slope of the line passing through the points (1, 3) and (4, 6).

SOLUTION We have

$$m = \frac{\text{[difference in } y\text{-coordinates]}}{\text{[difference in } x\text{-coordinates]}} = \frac{6-3}{4-1} = \frac{3}{3} = 1.$$

Thus, $m = 1$. Note that if we reverse the order of the points and use formula (1) to compute the slope, then we get

$$m = \frac{3-6}{1-4} = \frac{-3}{-3} = 1,$$

which is the same answer. The order of the points is immaterial. The important concern is to make sure that the differences in the x- and y-coordinates are formed in the same order. »» Now Try Exercise 7

The slope of a line does not depend on which pair of points we choose as (x_1, y_1) and (x_2, y_2). Consider the line $y = 4x - 3$ and two points (1, 1) and (3, 9), which are on the line. Using these two points, we calculate the slope to be

$$m = \frac{9-1}{3-1} = \frac{8}{2} = 4.$$

Now, let us choose two other points on the line—say, (2, 5) and (−1, −7)—and use these points to determine m. We obtain

$$m = \frac{-7-5}{-1-2} = \frac{-12}{-3} = 4.$$

The two pairs of points give the same slope.

Justification of Formula (1) Since (x_1, y_1) and (x_2, y_2) are both on the line, both points satisfy the equation of the line, which has the form $y = mx + b$. Thus,

$$y_2 = mx_2 + b$$
$$y_1 = mx_1 + b.$$

Subtracting these two equations gives

$$y_2 - y_1 = mx_2 - mx_1 = m(x_2 - x_1).$$

Dividing by $x_2 - x_1$, we have

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

which is formula (1). So the two definitions of slope lead to the same number. «

Let us now study four of the most important properties of the slope of a straight line. We begin with the **steepness property**, since it provides us with a geometric interpretation for the number m.

Steepness Property Let the line L have slope m. If we start at any point on the line and move 1 unit to the right, then we must move m units vertically in order to return to the line (Fig. 1 on the next page). (Of course, if m is positive, then we move up; and if m is negative, we move down.)